

# A Study of the H-dibaryon in Holographic QCD

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We study the H-dibaryon (uuddss) in holographic QCD for the first time. Holographic QCD is derived from a QCD-equivalent D-brane system ( $S^1$ -compactified D4/D8/ $\overline{D8}$ ) in the superstring theory via the gauge/gravity correspondence. In holographic QCD, all baryons appear as topological chiral solitons of Nambu-Goldstone bosons and (axial) vector mesons. In this framework, the H-dibaryon can be described as an  $SO(3)$ -type hedgehog state. We present the formalism of the H-dibaryon in holographic QCD, and perform the calculation to investigate its properties in the chiral limit.

**KEYWORDS:** H-dibaryon, holographic QCD, chiral soliton

## 1. Introduction

The H-dibaryon is a  $B = 2$   $SU(3)$  flavor-singlet bound state of uuddss. In 1977, R. L. Jaffe first predicted the existence of H-dibaryons from a group-theoretical argument of the color-magnetic interaction in the MIT bag model [1], and estimated the H-dibaryon mass to be  $M_H \simeq 2150$  MeV. Since the  $\Lambda\Lambda$  threshold is experimentally 2231 MeV, this model calculation implies that the H-dibaryon mass may be stable against the strong decay into  $\Lambda\Lambda$ . Several years later, the H-dibaryon was also investigated [2, 3] in the Skyrme model [4], where baryons are described as chiral solitons. This investigation suggests that the H-dibaryon mass is smaller than mass of two nucleons in the chiral limit. This result seems to support Jaffe's prediction.

However, the prediction of the low-mass H-dibaryon is excluded experimentally in 1991. Instead, the double hyper nuclei  ${}^6_{\Lambda\Lambda}\text{He}$  was found by Imai group [5]. In fact, the H-dibaryon is not stable against the strong decay, and no H-dibaryon bound state exists at least at the physical point. A possible reason of the theoretical failure is due to a large  $SU(3)$  flavor-symmetry breaking by the large s-quark mass,  $m_s \gg m_{u,d}$ . Then, the current interest is possible existence of the H-dibaryon as a resonance at the physical point.

Theoretically, there is also an interesting subject of the stability of the H-dibaryon at *unphysical* points such as  $SU(3)$  flavor-symmetric cases ( $m_u=m_d=m_s$ ). As a recent progress, lattice QCD calculations indicate the existence of the H-dibaryon at some “unphysical points”: the H-dibaryon seems to be stable at the  $SU(3)$  flavor-symmetric and large quark-mass region [6, 7].

Then, how about the H-dibaryon in the chiral limit of  $m_u=m_d=m_s=0$ ? Although the lattice QCD calculation is usually a powerful tool to evaluate hadron masses, it is difficult to take the chiral limit, since a large-size lattice is required for such calculations. Therefore, for the study of the chiral limit, some model approach [8] such as the Skyrme model [4] would be useful, instead of lattice QCD. Of course, it is desired to use a QCD-based model for the calculation.

In this paper, we study the H-dibaryon and its properties in the chiral limit using holographic QCD [9–11], a recently developed framework to analyze nonperturbative QCD. In particular, we investigate the H-dibaryon mass from the viewpoint of stability and “existence” of H-dibaryons in the chiral limit.

## 2. Holographic QCD

To begin with, we introduce holographic QCD [9, 10]. In the superstring theory, four-dimensional QCD can be constructed using an  $S^1$ -compactified D4/D8/ $\overline{\text{D8}}$ -brane system [10], which is called holographic QCD. This QCD-equivalent D-brane system consists of  $N_c$  D4-branes and  $N_f$  D8/ $\overline{\text{D8}}$ -branes, which give color and flavor degrees of freedom, respectively. In this construction, gluons and quarks appear as the fluctuation modes of 4-4, 4-8 and 4- $\overline{8}$  strings. This D-brane system has the  $\text{SU}(N_c)$  gauge symmetry and the chiral symmetry [10], and is basically equivalent to QCD in the chiral limit.

As is often used in holographic QCD, we take  $1/N_c$  and  $1/\lambda$  expansions, where the 't Hooft coupling  $\lambda \equiv N_c g_{\text{YM}}^2$  is expressed with the gauge coupling  $g_{\text{YM}}$ . In large  $N_c$  and large  $\lambda$ ,  $N_c$  D4 branes are replaced by a gravitational background via the gauge/gravity correspondence, and the strong-coupling gauge theory is converted into a weak-coupling gravitational theory [9].

In the D4-brane gravitational background, the D8/ $\overline{\text{D8}}$  brane system can be expressed with the Dirac-Born-Infeld (DBI) action,

$$S_{\text{D8}}^{\text{DBI}} = T_8 \int d^9 x e^{-\phi} \sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})}, \quad (1)$$

where  $F_{MN}$  is the field strength in the flavor space on the D8 brane, and  $T_8$ ,  $\phi$  and  $\alpha'$  are quantities defined in the superstring theory [10]. From this action, we derive the four-dimensional meson theory equivalent to infrared QCD at the leading order of  $1/N_c$  and  $1/\lambda$  [10, 11]. Here, we only consider massless Nambu-Goldstone (NG) bosons and the lightest vector meson “ $\rho$ -meson”, for the construction of low-energy effective theory, and finally derive the effective action in four-dimensional Euclidean space-time  $x^\mu = (t, \mathbf{x})$  [11]:

$$\begin{aligned} S_{\text{HQCD}} = \int d^4 x \{ & \frac{f_\pi^2}{4} \text{tr}(L_\mu L_\mu) - \frac{1}{32e^2} \text{tr}[L_\mu, L_\nu]^2 + \frac{1}{2} \text{tr}(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)^2 + m_\rho^2 \text{tr}(\rho_\mu \rho_\mu) \\ & - ig_{3\rho} \text{tr}\{(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)[\rho_\mu, \rho_\nu]\} - \frac{1}{2} g_{4\rho} \text{tr}[\rho_\mu, \rho_\nu]^2 + ig_1 \text{tr}\{[\alpha_\mu, \alpha_\nu](\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\} \\ & + g_2 \text{tr}\{[\alpha_\mu, \alpha_\nu][\rho_\mu, \rho_\nu]\} + g_3 \text{tr}\{[\alpha_\mu, \alpha_\nu](\beta_\mu, \rho_\nu) + [\rho_\mu, \beta_\nu]\} \\ & - ig_4 \text{tr}\{(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)(\beta_\mu, \rho_\nu) + [\rho_\mu, \beta_\nu]\} - g_5 \text{tr}\{[\rho_\mu, \rho_\nu](\beta_\mu, \rho_\nu) + [\rho_\mu, \beta_\nu]\} \\ & - \frac{1}{2} g_6 \text{tr}([\alpha_\mu, \rho_\nu] + [\rho_\mu, \alpha_\nu])^2 - \frac{1}{2} g_7 \text{tr}([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])^2 \}. \end{aligned} \quad (2)$$

Here,  $\rho_\mu(x) \equiv \rho_\mu^a(x) T^a \in \text{su}(N_f)$  denotes the  $\text{SU}(N_f)$  lightest vector meson ( $\rho$ -meson) field, and  $L_\mu$  is defined with the chiral field  $U(x)$  as

$$L_\mu \equiv \frac{1}{i} U^\dagger \partial_\mu U \in \text{su}(N_f), \quad U(x) \equiv e^{i2\pi(x)/f_\pi} \in \text{SU}(N_f), \quad (3)$$

where  $\pi(x) \equiv \pi^a(x) T^a \in \text{su}(N_f)$  is the NG boson field. The axial vector current  $\alpha_\mu$  and the vector current  $\beta_\mu$  are defined as

$$\alpha_\mu \equiv l_\mu - r_\mu, \quad \beta_\mu \equiv \frac{1}{2}(l_\mu + r_\mu), \quad (4)$$

with the left and the right currents,

$$l_\mu \equiv \frac{1}{i} \xi^\dagger \partial_\mu \xi, \quad r_\mu \equiv \frac{1}{i} \xi \partial_\mu \xi^\dagger, \quad \xi(x) \equiv e^{i\pi(x)/f_\pi} \in \text{SU}(N_f). \quad (5)$$

Thus, we obtain the effective meson theory derived from QCD in the chiral limit. This theory has just two independent parameters, e.g., the Kaluza-Klein mass  $M_{\text{KK}} \sim 1\text{GeV}$  and  $\kappa \equiv \lambda N_c / 216\pi^3$  [10], and all the coupling constants and masses in the effective action (2) are expressed with them [11].

Remarkably, in the absence of the  $\rho$ -meson, this effective theory reduces to the Skyrme model [4] in Euclidean space-time:

$$\mathcal{L}_{\text{Skyrme}} = \frac{f_\pi^2}{4} \text{tr}(L_\mu L_\mu) - \frac{1}{32e^2} \text{tr}[L_\mu, L_\nu]^2. \quad (6)$$

### 3. Topological Chiral Soliton Picture for the H-dibaryon in Holographic QCD

In general, large- $N_c$  QCD becomes a weakly interacting meson theory, and baryons are described as topological chiral solitons of mesons [12]. We note that the H-dibaryon is also described as a  $B=2$  chiral soliton in holographic QCD with large  $N_c$ , like the Skyrme model [2, 3]. We study the static H-dibaryon as a  $B=2$  chiral soliton in holographic QCD, using the ‘‘SO(3)-type hedgehog Ansatz’’

$$U(\mathbf{x}) = e^{i[(\mathbf{\Lambda} \cdot \hat{\mathbf{x}})F(r) + [(\mathbf{\Lambda} \cdot \hat{\mathbf{x}})^2 - 2/3]\varphi(r)]} \in \text{SU}(3)_f, \quad F(r) \in \mathbf{R}, \quad \varphi(r) \in \mathbf{R} \quad (r \equiv |\mathbf{x}|, \quad \hat{\mathbf{x}} \equiv \mathbf{x}/r) \quad (7)$$

with the  $B=2$  topological boundary condition [2, 3] of

$$F(\infty) = \varphi(\infty) = 0, \quad F(0) = \varphi(0) = \pi. \quad (8)$$

Here,  $\Lambda_{i=1,2,3}$  are the generators of the SO(3) subalgebra of  $\text{SU}(3)_f$ ,

$$\Lambda_1 = \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \Lambda_2 = -\lambda_5 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad \Lambda_3 = \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (9)$$

which satisfy

$$[\Lambda_i, \Lambda_j] = i\epsilon_{ijk}\Lambda_k, \quad \text{Tr}[(\mathbf{\Lambda} \cdot \hat{\mathbf{x}})^2 - 2/3] = 0, \quad (\mathbf{\Lambda} \cdot \hat{\mathbf{x}})^3 = \mathbf{\Lambda} \cdot \hat{\mathbf{x}}. \quad (10)$$

Note that  $U(\mathbf{x})$  in Eq.(7) is the general form of the special unitary matrix which consists of  $\mathbf{\Lambda} \cdot \hat{\mathbf{x}}$ . For the  $\text{SU}(3)_f$   $\rho$ -meson field, we use the SO(3) Wu-Yang-'t Hooft-Polyakov Ansatz, similarly in the  $B=1$  holographic baryon [11],

$$\rho_0(\mathbf{x}) = 0, \quad \rho_i(\mathbf{x}) = \epsilon_{ijk}\hat{x}_j G(r)\Lambda_k \in \text{so}(3) \subset \text{su}(3), \quad G(r) \in \mathbf{R}. \quad (11)$$

In this way, all the above treatments are symmetric in the (u, d, s) flavor space.

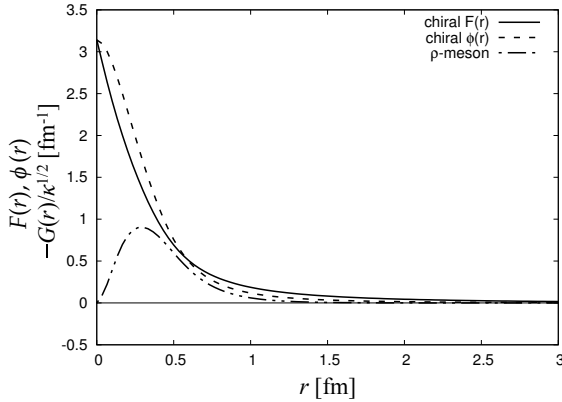
By substituting Ansätze (7) and (11) in Eq. (2), we derive the effective action to describe the static H-dibaryon in terms of the profile functions  $F(r)$ ,  $\varphi(r)$  and  $G(r)$  :

$$\begin{aligned} S_{\text{HQCD}} = \int d^4x \Big\{ & \frac{f_\pi^2}{4} \left[ \frac{2}{3} \varphi'^2 + 2F'^2 + \frac{8}{r^2} (1 - \cos F \cos \varphi) \right] \\ & + \frac{1}{32e^2} \frac{16}{r^2} [(\varphi'^2 + F'^2)(1 - \cos F \cos \varphi) + 2\varphi' F' \sin F \sin \varphi \\ & \quad + \frac{1}{r^2} \{(1 - \cos F \cos \varphi)^2 + 3 \sin^2 F \sin^2 \varphi\}] \\ & + \frac{1}{2} \left[ 8 \left( \frac{3}{r^2} G^2 + \frac{2}{r} G G' + G'^2 \right) \right] + m_\rho^2 [4G^2] + g_{3\rho} \left[ 8 \frac{G^3}{r} \right] + \frac{1}{2} g_{4\rho} [4G^4] \\ & - g_1 \left[ \frac{16}{r} \left\{ \left( \frac{1}{r} G + G' \right) \left( F' \sin \frac{F}{2} \cos \frac{\varphi}{2} + \varphi' \cos \frac{F}{2} \sin \frac{\varphi}{2} \right) + \frac{1}{r^2} G (1 - \cos F \cos \varphi) \right\} \right] \\ & - g_2 \left[ \frac{8}{r^2} G^2 (1 - \cos F \cos \varphi) \right] \\ & + g_3 \left[ \frac{16}{r^3} G \left\{ 3 \sin F \sin \frac{F}{2} \sin \varphi \sin \frac{\varphi}{2} + \left( 1 - \cos \frac{F}{2} \cos \frac{\varphi}{2} \right) (1 - \cos F \cos \varphi) \right\} \right] \\ & - g_4 \left[ \frac{16}{r^2} G^2 \left( 1 - \cos \frac{F}{2} \cos \frac{\varphi}{2} \right) \right] - g_5 \left[ \frac{8}{r} G^3 \left( 1 - \cos \frac{F}{2} \cos \frac{\varphi}{2} \right) \right] \\ & + g_6 [4G^2 (F'^2 + \varphi'^2)] + g_7 \left[ \frac{8}{r^2} G^2 \left\{ 3 \sin^2 \frac{F}{2} \sin^2 \frac{\varphi}{2} + \left( 1 - \cos \frac{F}{2} \cos \frac{\varphi}{2} \right)^2 \right\} \right] \Big\}. \quad (12) \end{aligned}$$

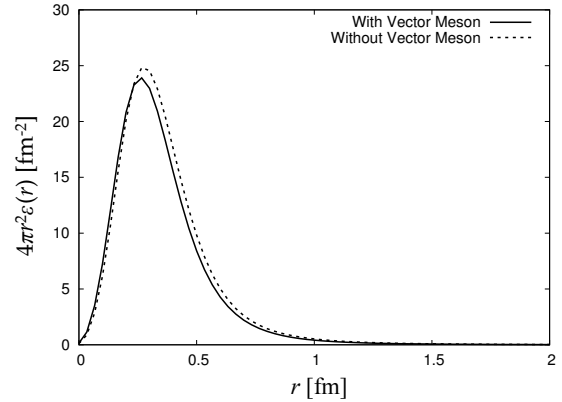
## 4. Numerical Results

Now, to investigate the H-dibaryon in the chiral limit, we perform the numerical calculation of the profiles  $F(r)$ ,  $\varphi(r)$  and  $G(r)$  to minimize the Euclidean effective action (12) under the boundary condition (8) [13]. Here, the two parameters, e.g.,  $M_{KK}$  and  $\kappa$ , are set so as to reproduce the pion decay constant  $f_\pi = 92.4\text{MeV}$  and the  $\rho$ -meson mass  $m_\rho = 776\text{MeV}$  [10, 11].

Figure 1 shows the chiral profiles  $F(r)$ ,  $\varphi(r)$  and the scaled  $\rho$ -meson profile  $G(r)/\kappa^{1/2}$  in the soliton solution of the H-dibaryon in holographic QCD. The H-dibaryon mass is estimated as  $M_H \simeq 1673\text{MeV}$ . We also calculate the energy density  $4\pi r^2 \varepsilon(r)$  in the H-dibaryon in Fig. 2, and estimate the root mean square radius in terms of the energy density as  $\sqrt{\langle r^2 \rangle} \simeq 0.413\text{fm}$ . For comparison, we note the  $B=1$  hedgehog-baryon mass and radius:  $M_{B=1}^{HH} \simeq 836.7\text{MeV}$  and  $\sqrt{\langle r^2 \rangle} \simeq 0.362\text{fm}$ . In fact, the H-dibaryon mass is almost equal to two  $B=1$  hedgehog-baryon mass,  $M_H \simeq 2.00 M_{B=1}^{HH}$ . Since the nucleon mass  $M_N$  is larger than the hedgehog mass  $M_{B=1}$  by the rotational energy, the H-dibaryon mass is smaller than mass of two nucleons (flavor-octet baryons),  $M_H < 2M_N$ , in the chiral limit.



**Fig. 1.** The chiral profiles  $F(r)$ ,  $\varphi(r)$  and the scaled  $\rho$ -meson profile  $G(r)/\kappa^{1/2}$  in the H-dibaryon as the  $SO(3)$ -type hedgehog soliton solution in holographic QCD.



**Fig. 2.** The energy density distribution  $4\pi r^2 \varepsilon(r)$  in the H-dibaryon (solid curve), and that without vector mesons (dashed curve) for comparison.

Finally, we investigate the vector-meson effect for the H-dibaryon. As the result, we find that the chiral profiles  $F(r)$  and  $\varphi(r)$  are almost unchanged and slightly shrink by the vector-meson effect, and the energy density also shrinks slightly, as shown in Fig. 2. We find, however, that about 100MeV mass reduction is caused by the vector-meson effect, and this mass reduction is due to the interaction between NG bosons and vector mesons in the interior region of the H-dibaryon.

## 5. Summary

We have studied the H-dibaryon (uuddss) as the  $B=2$   $SO(3)$ -type chiral soliton in holographic QCD for the first time. The H-dibaryon mass is estimated about 1.7GeV in the chiral limit, which is smaller than mass of two nucleons (flavor-octet baryons). In the H-dibaryon, we have found that, together with slight shrinkage of the chiral profile functions  $F(r)$ ,  $\varphi(r)$  and the energy density, about 100MeV mass reduction of the H-dibaryon is caused by the vector-meson effect.

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